Unified Natural Function Theory: Operator, Spectrum, and Field Dynamics

Abstract

We present a unified framework linking a symmetric mixed integral–differential operator with a dimensionless field theory.

1. Introduction

This work unifies operator theory and field dynamics into a single framework.

2. Mixed Operator

O[y] = a(D^2 + D^-2)y + b(D + D^-1)y + c y.

3. Spectrum

Λ(λ) = a(λ² + λ■²) + b(λ + λ■¹) + c.

4. Reduction

t = λ + 1/λ reduces the quartic to a quadratic.

5. Oscillatory Regime

|t| ≤ 2 gives λ = e^(iθ).

6. Hyperbolic Regime

|t| > 2 gives λ = e^u.

7. Geometry

Circular vs hyperbolic geometry.

8. Constants

Algebraic constants emerge from parameters.

9. Field Theory

Dimensionless field definitions.

10. Motion

Motion arises from ∇R.

11. Gradient

Drives motion.

12. Divergence

Controls expansion.

13. Curl

Controls rotation.

14. Memory

Adds persistence.

15. Unification

Spectral and field views unify.

16. Unity

Maps to unit circle.

17. Taxonomy

Defines natural functions.

18. Iteration

Generates attractors.

19. Applications

Simulation and physics.

20. Conclusion

Unified theory established.